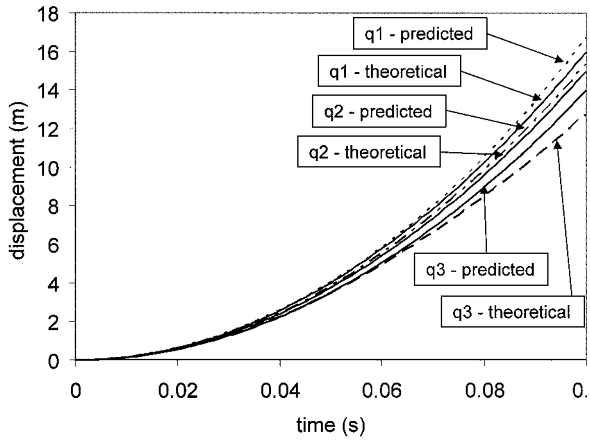

Fig. 2 Predicted and theoretical translations for test case C with 5% noise added using the 18-accelerometer configuration: ——, theoretical values and – – –, predicted values.

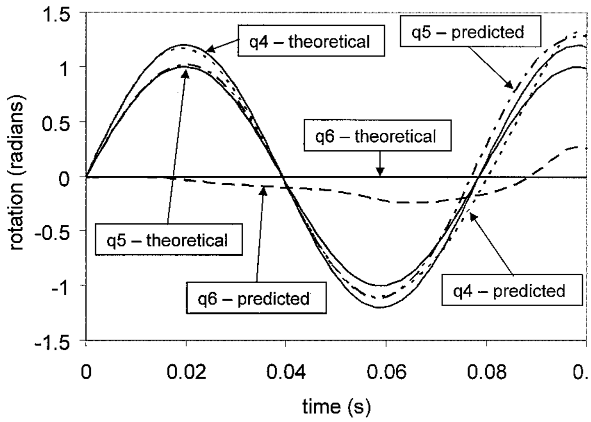

Fig. 3 Predicted and theoretical rotations for test case C with 5% noise added using 18-accelerometer configuration: ——, theoretical values and – – –, predicted values.

noise added and an 18-accelerometer configuration plotted over the 0.1-s time interval of interest. In previous studies, a suitable solution with six accelerometers to test case C was never found. However, several suitable 18-accelerometer configurations were found during the current study that would adequately reconstruct the trajectory over this time frame.

Conclusions

The overdetermined transducer formulation was shown to stabilize the solution and in most cases improve the accuracy. However, one might expect to see a point where adding additional accelerometers would provide diminishing returns. It was initially thought that 18 accelerometers would be more than necessary. However, if noise is high and time durations are long, the solution could benefit by using even more than 18 accelerometers. In practice, available data channels and accelerometer mounting locations would prevent this option.

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Pitch Autopilot Design Using Model-Following Adaptive Sliding Mode Control

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Nomenclature

A_m, B_m, A_p, B_p, B_c	=	known system matrices
$\boldsymbol{d}$	=	external disturbance vector
$\boldsymbol{e}$	=	error vector between the reference model and plant state vector
$\boldsymbol{g}_1$, $\boldsymbol{g}_2$, $\boldsymbol{g}_3$	=	gain vectors for the equivalent control
n_{zm}, n_{zp}, n_{zc}	=	pitch accelerations normalized by gravity, g
q_m, q_p	=	pitch rates, deg/s
s	=	switching function
u_{peq}, u_{puc}	=	equivalent and compensating control input
u_{puco}	=	compensating control input with ideal conditions
$\boldsymbol{X}_m$, $\boldsymbol{X}_p$	=	state vectors
x_{im}, x_{ip}	=	integrator states of the pitch acceleration error
$\boldsymbol{Z}_n$, s_n	=	normalized compensating vector and switching function
ΔA_p, ΔB_p	=	unknown model uncertainties
δ_m, δ_p, δ_c	=	elevator deflections, deg
ϕ, $\hat{\phi}$	=	adaptive gain and its estitimates for the compensating control input

Subscripts

c	=	command
m	=	reference model
p	=	plant

Introduction

THIS Note presents the model-following pitch autopilot design of a missile employing output feedback adaptive sliding mode control. In general, the classical proportional–integral (PI)-type pitch autopilot has been used for the control of the pitch acceleration, and it is composed of the output feedback of the pitch acceleration and angular rate and the integrator for eliminating the steady-state error.[1] In this Note, the classical PI-type reference model is constructed for the purpose of designing a model-following pitch autopilot, and the adaptive sliding mode control law is used to design an autopilot robust to model uncertainties and disturbances. Furthermore, output feedback is used in designing an adaptive sliding mode control law, and the model states are used for the unmeasured states instead of estimates.

The nominal models of the missile and actuator dynamics may contain the model uncertainties. Furthermore, the external disturbances are also applied to the plant. Hence, the control input of the adaptive sliding mode controller, composed of both the equivalent

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control input using the nominal parameters of the plant model and the adaptive control input compensating the model uncertainties and disturbances is proposed. The switching function defining the sliding surface is represented as the linear combination of the error between the states of the reference model and plant.[2,3] In the adaptive sliding mode control, like the indirect adaptive control,[4] the switching function asymptotically goes to zero so that the quasi-sliding mode is established.[5,6] The convergence of the switching function to zero does not mean that the pitch acceleration error between the reference model and plant goes to zero. Therefore, in this Note, the integrator for the error between the pitch acceleration of the reference model and plant is employed for eliminating the steady-state error. Simulation results are obtained that show asymptotic tracking, in spite of the presence of model uncertainties and disturbances. The sampled data systems are used for representing the plant and reference model because the control law is implemented in the digital computer.

Problem Formulation

The nonlinear missile dynamic equations considered here are taken from Ref. 7. These dynamics represent the pitching motion of a missile traveling at Mach 3 at an altitude of 6096 m. They do not correspond to any particular missile airframe. The nonlinear missile dynamics are

$$\dot{\alpha} = f\frac{\cos(\alpha/f)}{mV}F_z + q_p \tag{1}$$

$$\dot{q}_p = f\frac{M_y}{I_y} \tag{2}$$

where

D = reference diameter, 0.2286 m
F_z = normal force, $C_z QS$, lb
f = radians-to-degrees conversion, $180/\pi$
I_y = pitch moment of inertia, 247.44 kg · m^2
M_y = pitch moment, $C_m QSD$, ft · lb
m = mass, 242.02 kg
Q = dynamic pressure, 2534.27 N · m^2
S = reference area, 0.0409 m^2
V = speed, 947.71 m/s
α = angle of attack, deg

The normal force and pitch moment aerodynamic coefficients are approximated for the angle of attack α in the range of ± 20 deg as follows:

$$C_z = 0.000103\alpha^3 - 0.00945\alpha|\alpha| - 0.170\alpha - 0.034\delta_p \tag{3}$$

$$C_m = 0.000215\alpha^3 - 0.0195\alpha|\alpha| + 0.051\alpha - 0.206\delta_p \tag{4}$$

The missile tail fin actuator is modeled as the first-order system as transfer function

$$(\delta_p/\delta_c)(s) = 1/(s/\tau_a + 1) \tag{5}$$

where τ_a is the actuator time constant, $\frac{1}{150}$ s.

The pitch autopilot will be required to control the body's z axis (pitch) acceleration normalized by gravity:

$$n_{zp} = F_z/mg \tag{6}$$

where g is the acceleration of gravity.

The nonlinear state equations (1) and (2) are linearized about trim operating points ($M_y = 0$) to form linear state-space equations of the form[8]

$$\begin{bmatrix}\dot{\alpha}\\ \dot{q}_p\end{bmatrix} = \begin{bmatrix}\boldsymbol{Z}_\alpha & 1\\ M_\alpha & 0\end{bmatrix}\begin{bmatrix}\alpha\\ q_p\end{bmatrix} + \begin{bmatrix}\boldsymbol{Z}_\delta\\ M_\delta\end{bmatrix}\delta_p \tag{7}$$

$$\begin{bmatrix}q_p\\ n_{zp}\end{bmatrix} = \begin{bmatrix}0 & 1\\ (V/g)Z_\alpha & 0\end{bmatrix}\begin{bmatrix}\alpha\\ q_p\end{bmatrix} + \begin{bmatrix}0\\ (V/g)Z_\delta\end{bmatrix}\delta_p \tag{8}$$

where

$$\boldsymbol{Z}_\alpha = -\frac{\sin(\alpha/f)}{mV}F_z + f\frac{\cos(\alpha/f)}{mV}\frac{\partial F_z}{\partial\alpha} \tag{9}$$

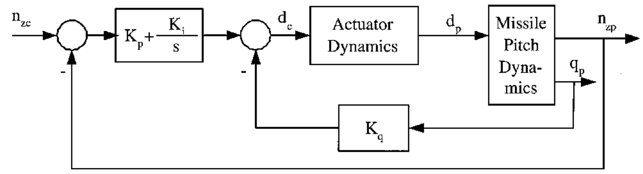

Fig. 1 Pitch autopilot using PI control.

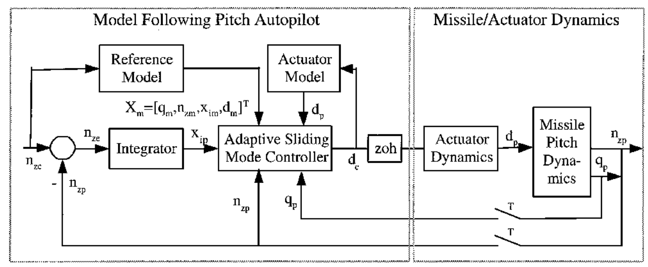

Fig. 2 Model following pitch autopilot using adaptive sliding mode control.

$$M_\alpha = \frac{f}{I_y}\frac{\partial M_y}{\partial\alpha} \tag{10}$$

$$\boldsymbol{Z}_\delta = f\frac{\cos(\alpha/f)}{mV}\frac{\partial F_z}{\partial\delta_p} \tag{11}$$

$$M_\delta = \frac{f}{I_y}\frac{\partial M_y}{\partial\delta_p} \tag{12}$$

The typical block diagram of the classical PI-type controller for a pitch autopilot can be seen in Fig. 1. (Ref. 1). The closed-loop system controlled by the classical PI-type controller is used as the reference model to design a model-following pitch autopilot. The state-space model of the reference model and plant are represented in sampled data system as

$$\boldsymbol{X}_m(k+1) = A_m\boldsymbol{X}_m(k) + B_m n_{zc}(k) \tag{13}$$

$$\boldsymbol{X}_p(k+1) = (A_p + \Delta A_p)\boldsymbol{X}_p(k) + (B_p + \Delta B_p)\delta_c(k) + B_c n_{zc}(k) + \boldsymbol{d}(k) \tag{14}$$

where $\boldsymbol{X}_m = [q_m, n_{zm}, x_{\text{im}}, \delta_m]^T$ and $\boldsymbol{X}_p = [q_p, n_{zp}, x_{\text{ip}}, \delta_p]^T$. It is assumed that the external disturbance vector has an upper bound, given by $\|\boldsymbol{d}(k)\| < v$, where $\|\cdot\|$ denotes the two-norm of a vector and v is a positive constant. This upper bound need not be known. Among the plant states, only the states q_p and n_{zp} are measured, and the states x_{ip} and δ_p are obtained from the model states.

Now, the purpose of the model-following pitch autopilot is that the plant output n_{zp} tracks the reference model's output n_{zm} asymptotically as

$$\lim_{k\to\infty}|n_{zm}(k) - n_{zp}(k)| = 0 \tag{15}$$

The structure of the model-following pitch autopilot of a missile employing output feedback adaptive sliding mode control is shown in Fig. 2.

Model-Following Adaptive Sliding Mode Control

The control input proposed in this Note comprises a nominal term u_{peq} for the known system and a compensating term u_{puc} to deal with model uncertainties and disturbances. This is represented as

$$\delta_c(k) = u_{\text{peq}}(k) + u_{\text{puc}}(k) \tag{16}$$

The error vector $\boldsymbol{e}(k)$ is defined as $\boldsymbol{e}(k) = \boldsymbol{X}_p(k) - \boldsymbol{X}_m(k)$. The switching function obtained as a linear combination of the error vector is represented as $s(k) = \boldsymbol{c}^T\boldsymbol{e}(k)$. The constant row vector $\boldsymbol{c}^T$ is designed so that the sliding mode is stable. The known nominal part of the plant is made use of in this design.[2,3]

In the absence of system uncertainties and disturbances, the equivalent control for sliding mode can be obtained from the condition for an ideal sliding mode given by $s(k+1)=0$ as

$$u_{\text{peq}}(k) = -\left(\boldsymbol{c}^T B_p\right)^{-1}\boldsymbol{c}^T[A_p\boldsymbol{X}_p(k) - A_m\boldsymbol{X}_m(k) + (B_c - B_m)n_{zc}(k)] = \boldsymbol{g}_1\boldsymbol{X}_p(k) + \boldsymbol{g}_2\boldsymbol{X}_m(k) + \boldsymbol{g}_3 n_{zc}(k) \quad (17)$$

If it is assumed that the model uncertainties ΔA_p and ΔB_p and disturbances $\boldsymbol{d}(k)$ of system (14) are known exactly, the sliding mode condition, together with the equivalent control, would immediately provide the compensating term, which is denoted by $u_{\text{puco}}(k)$, of the control input $\delta_c(k)$. Now consider $s(k+1)=0$, represented by

$$s(k+1) = \boldsymbol{c}^T[\boldsymbol{X}_p(k+1) - \boldsymbol{X}_m(k+1)] = \boldsymbol{c}^T\{(A_p + \Delta A_p)\boldsymbol{X}_p(k) + (B_p + \Delta B_p)[u_{\text{peq}}(k) + u_{\text{puco}}(k)] + \boldsymbol{d}(k) - A_m\boldsymbol{X}_m(k) + (B_{p2} - B_m)n_{zc}(k)\} = 0 \quad (18)$$

When the equivalent control input (17) is substituted into the sliding mode condition (18), $u_{\text{puco}}(k)$ can be determined as

$$u_{\text{puco}}(k) = -\left[\boldsymbol{c}^T(B_p + \Delta B_p)\right]^{-1}\boldsymbol{c}^T[\Delta A_p\boldsymbol{X}_p(k) + \Delta B_p u_{\text{peq}}(k) + \boldsymbol{d}(k)] = \boldsymbol{\phi}^T\boldsymbol{Z}(k) \quad (19)$$

where $\phi = [\phi_1^T \;\; \phi_2 \;\; \phi_3]^T$ and $\boldsymbol{Z}(k) = [\boldsymbol{X}_p(k)^T \;\; u_{\text{peq}}(k) \;\; 1]^T$ are column vectors and ϕ_1, ϕ_2, and ϕ_3 are defined as follows:

$$\phi_1^T = -\left[\boldsymbol{c}^T(B_p + \Delta B_p)\right]^{-1}\boldsymbol{c}^T\Delta A_p \quad (20)$$

$$\phi_2 = -\left[\boldsymbol{c}^T(B_p + \Delta B_p)\right]^{-1}\boldsymbol{c}^T\Delta B_p \quad (21)$$

$$\phi_3 = -\left[\boldsymbol{c}^T(B_p + \Delta B_p)\right]^{-1}\boldsymbol{c}^T\boldsymbol{d}(k) \quad (22)$$

Because model uncertainties $(\Delta A_p, \Delta B_p)$ and disturbances $\boldsymbol{d}(k)$ are unknown, an adaptive version of Eq. (19) is being proposed as

$$u_{\text{puc}}(k) = \hat{\phi}^T\boldsymbol{Z}(k) \quad (23)$$

$$\hat{\phi}(k) = \hat{\phi}(k-1) - \boldsymbol{Z}_n(k-1)s_n(k) \quad (24)$$

where $\hat{\phi}(k)$ is the estimate of $\phi(k)$. The normalized signals $\boldsymbol{Z}_n(k)$ and $s_n(k)$ are defined as

$$\boldsymbol{Z}_n(k) = \frac{\alpha\boldsymbol{Z}(k)}{1 + \|\boldsymbol{Z}(k)\|}, \qquad \alpha \in (0, 1] \quad (25)$$

$$s_n(k) = \frac{s(k)}{1 + \|\boldsymbol{Z}(k-1)\|} \quad (26)$$

where α is the parameter that adjusts the convergence rate of the adaptation algorithm.

Because $\boldsymbol{c}^T$ is designed to yield a stable sliding mode, stability of the closed-loop adaptive system is ascertained by guaranteeing sliding mode motion of the partially known system (14) through the equivalent control input (17) and adaptive correcting control input (23). However, because a quasi-sliding mode rather than a sliding mode is being established, the bounded input-bounded output notion of stability is being used to prove the global stability of the controlled system.

When the error due to the estimation algorithm (23) is designated as $\tilde{\phi}(k) = \hat{\phi}(k) - \phi(k)$, the properties of the estimation algorithm described by Eqs. (24–26) will be established. These properties suggest that the estimation error is bounded and converges asymptotically to zero. They are summarized in the following properties:

Property 1 is

$$\|\tilde{\phi}(k)\|^2 < \|\tilde{\phi}(0)\|^2$$

Property 2 is

$$\lim_{k\to\infty} \|\tilde{\phi}(k+1) - \tilde{\phi}(k)\|^2 = 0$$

Property 3 is

$$\lim_{k\to\infty} s_n^2(k) = 0$$

The proof of the preceding properties is based on the existence of Lyapunov function whose time difference is negative definite. See Ref. 5 for further details.

To analyze the stability of system (13–14) contolled by Eq. (16), the concept of the key technical lemma[9] is used. The concerned lemma without proof is stated as follows.

Lemma: If

$$\lim_{k\to\infty} \frac{|s(k)|}{1 + \|\boldsymbol{Z}(k)\|} = 0 \quad (27)$$

then, subject to

$$\|\boldsymbol{X}_p(k)\| \le C_1 + C_2 \max_{1\le i\le k} |s(k)|, \qquad 0 < C_1, \quad C_2 < \infty \quad (28)$$

it follows that $\boldsymbol{Z}(k)$ is bounded and

$$\lim_{k\to\infty} s(k) = 0 \quad (29)$$

This proves the existence of a quasi-sliding mode for the controlled system. This lemma, which establishes the boundedness of $\boldsymbol{X}_p(k)$ and $u_p(k)$, also ascertains the stability of the system.

Simulation Results

Simulation results obtained from applying the model-following adaptive sliding mode control to the pitch acceleration control of the missile are presented in this section. If the nonlinear state equations are linearized for an angle of attack of −5 deg, the state-space equations of the missile in the pitch plane are given by

$$\begin{bmatrix}\dot{\alpha}\\ \dot{q}_p\end{bmatrix} = \begin{bmatrix}-0.9043 & 1\\ -81.2550 & 0\end{bmatrix}\begin{bmatrix}\alpha\\ q_p\end{bmatrix} + \begin{bmatrix}-0.1205\\ -130.8976\end{bmatrix}\delta_p \quad (30)$$

$$\begin{bmatrix}q_p\\ n_z\end{bmatrix} = \begin{bmatrix}0 & 1\\ -1.5405 & 0\end{bmatrix}\begin{bmatrix}\alpha\\ q_p\end{bmatrix} + \begin{bmatrix}0\\ -0.2040\end{bmatrix}\delta_p \quad (31)$$

The discrete state-space model of the reference model and plant using Eqs. (30) and (31) are obtained by discretization with sampling period of 0.01 s, where the state vector includes the states of the integrator and the actuator dynamics. The reference model was constructed by using the PI control parameters, $K_q = -0.1908$, $K_p = 0.7973$, and $K_i = 4.5246$, in Fig. 1. The plant state of the integrator was constructed by using $K_i = 3.1672$. To ensure stability of the system in sliding mode, the matrix $\boldsymbol{c}^T$ was so designed that it places all of the poles of the system in sliding mode at 0.9 in the z domain. The row vector $\boldsymbol{c}^T$ was obtained as

$$\boldsymbol{c}^T = [-0.2669 \quad 1.2452 \quad -1.5452 \quad 1.3262] \quad (32)$$

The feedback gains of the equivalent control input $u_{\text{peq}}(k)$ were then deduced as

$$\boldsymbol{g}_1 = [0.2855 \quad -1.1585 \quad 1.5452 \quad -0.6581]$$

$$\boldsymbol{g}_2 = [-0.0808 \quad 0.3720 \quad -0.5102 \quad 0.4825]$$

$$\boldsymbol{g}_3 = 0.8524 \quad (33)$$

To test the robustness property in sliding mode, the actuator time constant τ_a was doubled. Furthermore, the external disturbance, $\boldsymbol{d}(k) = [1/57.3 \;\; 1/10 \;\; 0 \;\; 0]^T$, which were biases in the measurements, was applied to the system.

Figure 3 shows the simulation results of the pitch acceleration for the step and time-varying commands. From the results, the controller for $\alpha = -5$ deg proved adequate for the α range of ±10 deg. Although the plant is a nonminimum phase system, the unstable pole-zero cancellation in the model reference adaptive control system does not occur because the pole assignment control scheme has been used. Figures 3 reveal that the quasi-sliding mode was established and also show that the tracking capability of the controller and the robustness to the model uncertainties and disturbances were achieved.

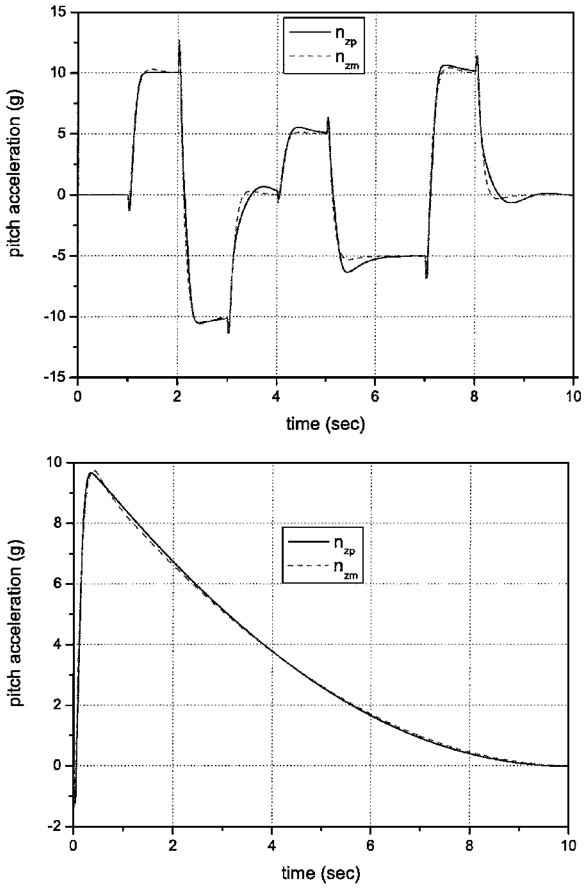

Fig. 3 Simulation results for the step and time-varying commands.

Conclusions

A model-following pitch autopilot using adaptive quasi-sliding mode control for a sampled-data system with model uncertainties and disturbances has been presented. The unknown parameters, which need not satisfy the matching conditions and the unknown disturbances and whose upper bound need not be known, are compensated for by applying an on-line adaptive algorithm. The proposed controller shows the asymptotic tracking of the pitch acceleration of the reference model and is able to provide robust performance of the system to the model uncertainties and disturbances.

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Hybrid Fuzzy Sliding-Mode Control of an Aeroelastic System

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Introduction

GROUND and flight tests of several advanced high-performance aircraft have exhibited a variety of nonlinear aeroelastic responses, including limit-cycle oscillation (LCO) and even instability.[1,2] Recently, a series of papers have considered the control of the aeroelastic system.[2–5] Most of the preceding papers consider the single-input, single-output aeroelastic control problem, that is, flap deflection has been used to control the pitch angle, and the plunge displacement has been shown to be asymptotically stable without control[2,5]; or the plunge displacement has been controlled by the flap deflection, and the pitch angle has been shown to be asymptotically stable without control.[4,5] Moreover, most of these design methods require a system model and complex design procedures. However, the modeling of an aeroelastic system is a work of approximation, and the precise model of an aeroelastic system can be difficult to formulate.

Fuzzy control using linguistic information can model the qualitative aspects of human knowledge. Fuzzy control also possesses several advantages such as robustness, model-free and rule-based algorithm.[6] However, the huge amount of fuzzy rules makes the analysis complex. Some researchers have proposed fuzzy sliding-mode control design methods to reduce the fuzzy control rules.[7,8] This Note proposes a hybrid fuzzy sliding-mode control (HFSMC) scheme for a nonlinear aeroelastic system. The aeroelastic system can be represented as two second-order subsystems that represent the plunge and pitch motions, respectively. By using the sliding-mode control, each subsystem can be controlled in terms of a corresponding sliding surface. A hybrid sliding surface, which includes two subsystems' information, is defined to generate a control effort to make the state trajectories of both subsystems move toward their sliding surface and then simultaneously approach zeros. A comparison between an adaptive control and the proposed HFSMC for an aeroelastic system is presented. Simulation results indicate that the proposed HFSMC can achieve superior control responses for the simultaneous control of plunge and pitch motions.

Nonlinear Aeroelastic Control System

A prototypical aeroelastic wing section is shown in Fig. 1. Define the state variables and the control input as

$$\boldsymbol{x} = [x_1 \quad x_2 \quad x_3 \quad x_4]^T = [h \quad \dot{h} \quad \alpha \quad \dot{\alpha}]^T \tag{1}$$

$$u = \beta \tag{2}$$

where h is the plunge displacement, α is the pitch angle, and β is the flap deflection. The equations of the motion can be written as

$$\dot{x}_1 = x_2, \qquad \dot{x}_2 = f_1(\boldsymbol{x}) + g_1 u$$
$$\dot{x}_3 = x_4, \qquad \dot{x}_4 = f_2(\boldsymbol{x}) + g_2 u \tag{3}$$

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